

*Citation for published version:*

Darling, J, Patel, A & Tilley, D 2011, 'Prediction of the pressure-flow characteristic of a Belleville washer-based damper valve', *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 225, no. 8, pp. 1033-1043. <https://doi.org/10.1177/0954407011403977>

*DOI:*

[10.1177/0954407011403977](https://doi.org/10.1177/0954407011403977)

*Publication date:*

2011

*Document Version*

Peer reviewed version

[Link to publication](#)

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# **The Prediction of the Pressure/Flow Characteristic of a Belleville Washer Based Damper Valve**

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**Keywords:** Bellville washer; Damper valve; Automotive suspension systems; pressure/flow characteristics; CFD; simulation.

## **Abstract**

Flow control valves used in automotive suspension dampers are almost universally based upon the restriction of flow using Belleville washers. These washers can best be described as a non-flat washer of conical shape and uniform thickness. They are also known as disc springs and are commonly used for load bearing in which their compactness and ability to produce a wide range of load-deflection characteristics provides an alternative to conventional coil springs.

Due to the non-linear behaviour of Belleville washers, and the complex flow paths in an automotive damper valve, it is difficult to predict the resulting pressure-flow characteristic. As a result, damper valve design is often considered to be a ‘black art’ best carried out by an experienced engineer using an experimental based trial and error approach. Clearly, without the aid of an analytical tool to predict the behaviour of prototype systems it is difficult for the designer to fully exploit the functionality of a particular design.

This paper describes theoretical and experimental studies undertaken to investigate the pressure-flow characteristic of a Belleville washer based damper valve. Existing load/deflection theory was extended to account for the hydraulic pressure acting on the surfaces of the washer. However, poor agreement was obtained between the measured pressure/flow data using a uniform pressure distribution across the washer diameter. For this reason the model was developed further to include the effect of a non-uniform pressure distribution within the valve. Computational Fluid Dynamics software was used to predict the pressure regime within the valve and this, in turn, was used to calculate the forces acting on the surface of the Belleville washer. The improved model was found to match the experimental behaviour with good accuracy, both for a range of spring pre-load conditions and for applications where several washers are stacked up in parallel. However further work would be required to develop a reliable simulation tool as an element of trial and error was necessary to estimate some simulation parameters.

## **1. Introduction**

A Belleville washer, or disc spring, is a non-flat washer formed with a slight conical shape that produces a spring characteristic when loaded. Washers of this type are widely used in a range of engineering applications, primarily where a compact spring is required for load bearing. Different non-linear load-deflection characteristics can be obtained by changing the ratio of the washer free height ( $h$ ) and thickness ( $t$ ) as shown in figure 1 [1]. When the ratio is sufficiently high, it is possible to obtain both positive and negative spring rates from a single washer. The load capacity and load deflection can also be varied by using multiple washers. For example, stacking four Belleville washers in

parallel causes the force/unit displacement to increase by a factor of four compared to a single washer, whereas stacking the same number of washers top to tail in series causes the displacement/unit force to increase by a factor of four.

The force-deflection characteristic of Belleville washers was first modelled by Almen and Laszlo [2] who assumed that when subjected to axial loads, the radial stresses in a disc spring were negligible and the cross-section does not distort but rotates about a neutral circumferential axis. Their force ( $P$ ) and displacement ( $\delta$ ) expression is presented below in equation 1.

$$P = \frac{\delta E}{(1-\nu^2)a^2 C_1} \left[ t(h-\delta) \left( h - \frac{\delta}{2} \right) + t^3 \right] \quad (1)$$

Where  $C_1$  is dependent upon the ratio ( $\alpha$ ) of the external radius ( $a$ ) to the internal radius ( $b$ ). Other variables are included in the notation.

$$C_1 = \frac{1}{\frac{\pi}{6} \left( \frac{\alpha}{\alpha-1} \right)^2 \log_e \alpha}$$

This equation was developed further by Curti and Orlando [3] who took account of radial stress to develop the expression shown in equation 2.

$$P = \frac{\delta E}{a^2} \left[ t(h-\delta) \left( h - \frac{\delta}{2} \right) C + Dt^3 \right] \quad (2)$$

where

$$C = \frac{2\pi}{(1-\nu)} \frac{\alpha^2}{(\alpha-1)^3} \left[ \frac{1+\alpha}{2} + \left( \frac{\nu}{1+\nu} \right) \left( \frac{\alpha^{(\nu+1)}-1}{1-\alpha^\nu} \right) \right] \quad \text{and} \quad D = \frac{\pi}{6} \frac{a}{r_n} \left( \frac{\alpha}{\alpha-1} \right)$$

In reality, for steel springs with a Poisson's ratio of 0.3 the radial stresses are small and Almen's model is generally considered sufficient.

In an automotive damper, Belleville washers are used to restrict the flow of an oil-based fluid in a valve that separates the damper piston from an expansion chamber, thereby dissipating energy in the form of heat. The principle of operation is illustrated by figure 2 in which a Belleville washer is pre-loaded on to the face of a plate using a bolt. For this arrangement, the hydraulic pressure acts on the washer's lower face and in order for flow to pass through the valve, the pressure difference acting across the washer must be sufficient to lift the edge of the washer from the plate. The flow passing through the radial orifice generated as a result of the washer 'cracking open' is determined by the pressure drop and the valve lift. The energy dissipated by the valve is the product of pressure and flow and is responsible for generating the damping effect and is characterised at the suspension as a relationship between force and velocity. The non-linear spring characteristic, the opportunity to pre-load the spring and the number and size of passages connecting the damper cylinder chamber and valve are all parameters that help the designer to vary the damping characteristic and make this design much more versatile than a simple fixed orifice. Currently very extensive experimental testing is used for damper valve development and a prediction tool is not available to help the designer to optimise the characteristic.

The aim of this paper is to develop a simulation model of the disc spring arrangement shown in figure 2. Here the flow is restricted to the outer circumference, ie no flow passes through the inner diameter.

## 2. Uniform Pressure Model for Damper Valve Flow

Equation (1) is based upon the assumption that the Belleville washer is supported at its internal and external diameters  $b$  and  $a$ , respectively as shown in figure 3. This is not the case when the washer is used as a damping element as the hydraulic pressure will be acting over the face of the washer. However, it is possible to represent the uniform pressure distribution as an equivalent force ( $P_{oil}$ ) acting at a radius ( $r_{eq}$ ) In this case, the moments due to the point and distributed loadings were determined as follows:

$$P_{oil}r_{eq} = p \int_b^a 2\pi r^2 dr = \frac{2}{3} \pi p (a^3 - b^3) \quad (3)$$

The total force ( $P_{oil}$ ) generated by the uniform pressure ( $p$ ) is given by:

$$P_{oil} = pA \quad \text{where} \quad A = \frac{\pi}{4} ((2a)^2 - (2b)^2) \quad (4)$$

Combining eqns.(3) and (4) gives;

$$r_{eq} = \frac{1}{3} \frac{((2a)^3 - (2b)^3)}{((2a)^2 - (2b)^2)} \quad (5)$$

Since the slope of the washer is small, the washer can be treated as a flat disc with  $P_{oil}$  acting at radius  $r_{eq}$  being reacted by a force  $P$  acting at radius  $b$ . The forces acting on the washer generate a moment  $M_c$  about the neutral point  $c$ , where;

$$M_c = P_{oil}(r_{eq} - c) + P(c - b) \quad (6)$$

Given that  $P$  is the reaction force to  $P_{oil}$ , eqn.(6) can be simplified to;

$$M_c = P_{oil}(r_{eq} - b) \quad (7)$$

For a segment of a washer which is subject to a load  $P$  applied at the inner and outer diameters the moment about the neutral point is:

$$M_c = P(a - b) \quad (8)$$

By combining eqns. (8) and (1) the following solution is obtained relating the force generated by the hydraulic fluid to the Belleville washer deflection;

$$P_{oil} = \frac{\delta E}{(r_{eq} - b)(a - b)(1 - \nu^2)a^2 C_1} \left[ t(h - \delta) \left( h - \frac{\delta}{2} \right) + t^3 \right] \quad (9)$$

The flow rate ( $Q$ ) passed by the damper valve at a particular pressure differential pressure,  $\Delta p$ , acting across the washer can be determined using the standard hydraulic orifice equation as follows:

$$Q = C_q A_{flow} \sqrt{\frac{2\Delta p}{\rho}} \quad (10)$$

where the annular flow area ( $A_{flow}$ ) between the washer external diameter ( $a$ ) and the plate is given by:

$$A_{flow} = 2\pi a \delta \quad (11)$$

### 3. Experimental Testing of Damper Valve

To validate the damper valve model, an experimental test rig was built incorporating the Belleville washer installation arrangement previously shown in figure 2. The load bolt both locates each washer and provides an adjustable pre-load that controls the washer ‘cracking pressure’. The two seals shown prevent flow through the central section, ensuring that the flow passes through the orifice formed at the external circumferential diameter. A schematic diagram of the flow and measurement circuit is shown in figure 4.

Tests were undertaken using a range of different washer geometries including those shown in Table 1.

Large variations in the experimental results were initially experienced when measuring the washer pressure/flow characteristics. However, good repeatability was obtained when the following test methodology was employed:

1. The components were allowed to reach room temperature.



2. Each component was dipped into a bath of oil to ensure good lubrication of the threads and surfaces.
3. The test washer was placed on the steel plate followed by a centring washer and seals.
4. The load bolt was then inserted and tightened to a torque of 1 Nm.
5. The test plate and the associated components were then inserted into the test housing which was then connected to the rig.
6. HLP32 hydraulic fluid was then circulated through the rig to allow the components to heat up until a steady state test temperature of 50°C was obtained.
7. The flow rate was then set to a minimum value of 20 L/min and the pressure drop noted.
8. The flow rate was then increased in 5 L/min increments and the pressure drop noted. This was repeated at flow rates up to 80 L/min.
9. The flow rate was then reduced in increments of 5L/min and the pressure drop noted.
10. The test plate was then removed from the rig and the bolt and washer were removed and allowed to cool down to room temperature.
11. The procedure was then repeated for pre-load torques in increments of 1Nm up to 5Nm.

The measured results obtained for a single, type A washer, presented in figure 5, show a near linear relationship between pressure and flow with some hysteresis, which is thought to be caused by friction at the inner diameter of the Belleville washer. The 5Nm

pre-load was thought to have led to the washer ‘bottoming out’ and for this reason the 5Nm results are not representative of the true valve behaviour.

Tests were also conducted using a single, and a parallel stack of two and three type B washers. The results presented in figure 6 indicate, not surprisingly, that the greater the number of washers the more restrictive the valve. In addition, as the number of washers are increased the larger the hysteresis.

#### 4. Simulation Study

A steady state simulation model of the disc valve was developed using the simulation environment *Bath/p* [5]. This package has been specifically designed for the dynamic simulation of fluid power and mechanical systems and was eminently suitable for the simple hydraulic circuit described here. The initial simulation model was based on a uniform pressure acting on the underside of the Belleville washer and a variable pre-load force from the restraining bolt. In addition, a variable flow coefficient  $C_q$  was used based on the work of Lichtarowicz on nozzle/flapper valves [6].

The approach used to determine the washer deflection and flow rate at a particular system pressure is illustrated in figure 7. This was based on using a look-up table to determine the washer deflection at a particular pressure, based on the forces determined from eqn. (9) for a suitable range of deflections. The look-up table was used to determine the washer pre-compression  $x_{pre}$  produced by the pre-load force and the deflection  $x$  resulting from the pressure force. The valve opening was then based on the difference between the two values ie  $\delta = x - x_{pre}$  with the valve closed when  $\delta < 0$ .

A comparison between the measured results obtained at 1Nm and 2Nm and the predicted behaviour for a single type A washer is shown in figure 8. Although it was not possible to measure the characteristic with zero pre-load, the predicted response is included on the figure. This shows a pressure drop of around 12bar at the maximum flow rate of 80L/min. As the tightening torque is increased, the predicted pressure/flow curves become flatter and show poor agreement with the experimental results. The pre-load force settings to obtain the cracking pressures will be discussed later in Section 6.

The force/displacement characteristics for the washers being used in the study were measured directly using an Instron materials test machine. The results obtained are compared with the theoretical curve in figure 9. The measured characteristic varies slightly depending on the particular washer being used and the results show some stiffening at the higher deflections which does not occur on the predicted curve. However, the deviation between the measured and predicted curves is not sufficient to cause the large disparity between the measured and simulated pressure drops indicated in figure 8.

The possible causes for the poor agreement were thought to be due to the pressure distribution acting on the washer, frictional effects and asymmetric opening of the washer. Frictional effects were shown to be of secondary importance as the introduction of these additional forces into the simulation model had little influence upon the valve performance. Similarly, it was considered that any asymmetric opening would have little effect on the total flow area and it was unlikely that such an opening would have a significant influence on valve performance.

The static pressure acting on the washer will be reduced due to the action of fluid flow. If this effect is significant, the assumption of uniform pressure acting on the washer face will be inappropriate and it will be necessary to take into account the pressure profile acting on the underside of the washer. As it was not practical to determine the distribution experimentally, Computational Fluid Dynamics (CFD) was used to assess the variation in the static pressure acting on the washers.

## **5. CFD Study**

In a CFD study, the fluid domain is subdivided into a collection of elements or control volumes. The solution procedure is iterative and the quality of the final solution is determined by the magnitude of the residual errors. The quality of the solution is not only defined by the residuals but also by the quality of the mesh. Since the governing equations are based on Taylor's expansion and truncation series, the final solution is heavily dependent on the number of elements used; To ensure that the chosen number of mesh elements does not affect the final solution it is standard practice to conduct a 'Grid Independency' study in which the mesh is refined until the influence on the final solution is negligible.

CFD methods are applicable to a variety of engineering disciplines including both large body simulations and small regions of flow, such as those found in poppet and spool valves. With a lack of publications on the use of Belleville washers as dampers, published work on the use of CFD to predict flow forces on poppet and spool valves were used for guidance.

The CFD study was undertaken using Ansys CFX version 5 based upon a 2D static model of fluid flow. In order to determine an appropriate mesh density the work of other fluid power researchers, including Vaughan et al [7], Johnston [8] and Borghi [9,10] was used .

The 2D axi-symmetric model used for the CFD simulations was based on a unity mesh depth, corresponding to an included angle of  $5^\circ$ . Since the underside of the washer and the exit were of interest, the geometry was simplified with the axial locating washer and bolt omitted. A grid independence study was undertaken and rather than using a uniform grid, a high density mesh was used around the inlet and exit areas of interest and a relatively coarse mesh was used for the upstream and downstream volumes. It was recognised that the exit length was of importance due to the likelihood of recirculation that would have an impact upon the predicted pressure profile. It was found that an exit length of 20 diameters provided a good balance between the simulation convergence time and accuracy.

The fluid was assumed to be Newtonian, isothermal and incompressible, having a density of  $875\text{kg/m}^3$  and a kinematic viscosity  $15\text{cSt}$ . No slip was included at the walls, and the pressure at the outlet was taken to be atmospheric. The fluid domain was assumed to be turbulent and modelled using the  $k$ - $\epsilon$  model [7]. Although this turbulence model can introduce errors, especially in zones of recirculation, the results presented by Yang [11] indicated that these had little effect on CFD predictions similar to those presented here. Based on the work of Borghi [9,10] the turbulence of the inlet jet was set to 5% and convergence was considered to have taken place when the maximum residual error was less than  $10^{-4}$ .

Steady-state simulations were undertaken at flow rates of 20, 40, 60 and 80L/min and washer deflections that were fixed between 0.1 and 1mm. Table 2 indicates the predicted pressure differential obtained at these set conditions. In this way, it was possible to estimate the values for the flow coefficient  $C_q$  and the flow number

Figure 10 compares the values for the flow coefficients obtained from CFD, simulation and from experiment when plotted against the flow number [4]. Although the values obtained for the CFD flow coefficients tend to reduce with increasing flow number, the agreement obtained with the experiment and simulated results is considered to be acceptable. Since the best agreement for the CFD predictions were obtained at 0.1 and 0.3mm openings, these displacements will be used to examine the velocity and pressure distributions.

Examination of the predicted velocity profile beneath the washer shown in figure 11 indicates two distinct regions for flow: an inner region of low or zero velocity and an outer region of high velocity. For the 34mm diameter washer used, the neutral axis was located at a radius of around 11mm and the localised drop in static pressure outside of the neutral point would result in less pressure to open the disc valve or alternatively a greater differential pressure is required for a given flow rate. In other words, the pressure distribution acting on the washer makes the valve more restrictive to flow. A more detailed representation of the velocity profile beneath the washer is shown in figures 12 and 13. When the opening at the outlet is 0.1mm (figure 12) the velocity profile and associated pressure is almost uniform. Increasing the opening to 0.3mm (figure 13) produces a higher velocity at the washer surface and a lower velocity at the fixed plate.

The results of figure 14 illustrate the variation of pressure along the Bellville washer radius obtained from the CFD study at a flow rate of 80L/min and deflection of 0.5mm. This shows an increase in the dynamic pressure at a radius of 10mm, although this increased to around 13mm at other conditions. The high velocity at the outer radius results in a significant reduction in the static pressure, thus confirming the assumption of uniform pressure to be inappropriate.

## 6. Modified Simulation Model

Based on the findings obtained from the CFD study, the analysis used in the simulation model was modified to take account of the pressure variation acting on the washer. For simplicity, the static pressure was divided into two regions; a constant pressure region and a region where the pressure reduced due to the increased velocity pressure. The dynamic pressure at the outer diameter (  $P_{Dynamic}$  ) was calculated using the following dynamic pressure term:

$$P_{Dynamic} = \frac{k\rho}{2} u_{mean}^2 \quad (12)$$

where  $u_{mean}$  is the average flow velocity,  $\rho$  the fluid density and  $k$  is a user defined factor to take account of the velocity acting at the washer surface.

The reduction in static pressure was taken into account in the simulation model by representing the washer surface as annular rings extending from the inner to outer radii. The velocity pressure acting on each elemental ring was then found by first determining the mean velocity in the gap created between the washer and plate. In this way, it was

possible to estimate the total force and moment by simply summing the forces and moments acting at each element. This then allowed the radius  $r_{eq1}$  at which the velocity pressure force  $P_{vp}$  acts to be determined numerically in a similar way to the analytical solutions obtained previously for equations (3) to (5). The total moment acting about the neutral point could then be determined using a modified version of equation (6) as follows;

$$M_c = P_{oil}(r_{eq} - c) - P_{vp}(r_{eq1} - c) + (P_{oil} - P_{vp})(c - b) \quad (13)$$

Including the velocity pressure term requires data relating to both the radius  $r_{eq1}$  and the corresponding  $k$  factor. Suitable values for these parameters were obtained from the simulation model using ‘trial and error’ to obtain the best fit to the experimental results shown earlier. In addition, pre-load forces are required to achieve the cracking pressure settings. Although it is possible to estimate the pre-load force induced in the central loading bolt at a particular tightening torque [12], the analysis is based on an assumed value for the coefficient of friction at the threads. Alternatively, the relationship between tightening torque ( $T$ ) and induced load ( $F$ ) can be approximated using the following simple equation:

$$T = Fk_a d \quad (14)$$

where  $k_a$  is factor depending on the thread material and  $d$  is the nominal bolt diameter.



Using  $k_a = 0.2$  in eqn. (14), gave an induced load of around 500N at a tightening torque of 1Nm, compared to a value of 450N obtained from the simulation.

Figure 15 shows a much improved comparison between the measured and simulated responses obtained using the modified model for the type A washer at different torque settings. This result was obtained using the values for  $r_{eq1}$ ,  $k$  and pre-load included in table 3. The 4Nm characteristic was obtained by assuming that the washer stiffness increased at high displacements, in-line with the experimental results shown in fig. 9.

In order to achieve the agreement shown in fig. 15, it was necessary to include a small increase in the value for  $r_{eq1}$  (11mm to 13mm) as the torque was raised from 1Nm to 4Nm. This was accompanied by a large reduction in the value of  $k$  (2.2 to 0.75) to account for the change in velocity distribution as the washer was compressed. As  $k$  has a significant influence on the predicted pressure/flow characteristic some experimental test work will still be necessary when designing a damper valve, particularly when different sizes of washer are to be used.

A similar approach was used to predict the pressure/flow characteristics for the stacked arrangement of type B washers shown in fig. 16. In this case, the simulation model was based on the parametric data included in table 4 and the results obtained show good agreement with the measured characteristics for increasing flow rate. However, the current model does not account for the hysteresis loop introduced when the flow rate is reduced.

## **7. Conclusions**

An experimental and simulation study of the pressure/flow characteristics of a Belleville washer type valve used in vehicle dampers is presented. It is shown that a simulation model based on uniform pressure beneath the Belleville washer was flawed and produced poor agreement with experiment. A number of mechanisms were investigated to account for this poor agreement and a non-uniform pressure distribution was identified as the principal cause. A CFD study was undertaken to predict the pressure profile acting on the washer for a range of valve openings and flow rates and the high velocity region at the outer diameter of the washer was found to be significant. When the pressure reductions associated with the high velocities were introduced into the simulation model good agreement was obtained between measurement and prediction for positively increasing pressures for single and multiple washers. However, due to friction effects the agreement was less good for decreasing pressures where significant hysteresis was evident on the experimental results. Despite good agreement between simulation and experiment it is still argued that further work is necessary in order to develop techniques that will help predict the variation of the velocity pressure factor  $k$ .

### **Acknowledgements**

The authors would like to thank the University of Bath and Horstman Defence for their financial support of this research.

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## Notation

$a$	Washer external radius
$A$	Surface area
$A_{flow}$	Flow area
$b$	Washer internal radius
$c$	Neutral point radius
$C_q$	Flow coefficient
$C, C_I$	Factors
$d$	Bolt diameter
$D$	Factor
$E$	Young's Modulus
$F$	Induced load
$h$	Washer free height
$k$	Velocity pressure factor
$k_d$	factor
$M_c$	Moment acting on about neutral point
$p$	Pressure
$P$	Axial load
$P_{Dynamic}$	Dynamic pressure
$P_{oil}$	Hydraulic pressure force
$P_{vp}$	Velocity pressure force
$Q$	Flow rate
$r$	Radius

$r_{eq}$	Equivalent radius for pressure force
$r_{eq1}$	Equivalent radius for velocity pressure force
$t$	Washer thickness
$T$	Tightening torque
$u_{mean}$	Fluid velocity
$x$	Washer displacement due to pressure force
$x_{pre}$	Washer pre-compression due to pr-load
$\alpha$	External radius to internal radius ratio
$\Delta p$	Pressure drop
$\delta$	Washer vertical deflection
$\nu$	Poisson's ratio
$\rho$	Hydraulic fluid density

## List of Tables

**Table 1** – Belleville washer dimensional data

**Table 2** - Predicted pressure differential (bar) obtained from CFD simulations

**Table 3** – Parametric data for type A washer

**Table 4** – Parametric data for type B washers

## List of Figures

**Figure 1** – Effect of washer free height/thickness ratio on Belleville on load-deflection characteristic

**Figure 2** - Disc spring used as a damper element

**Figure 3** – Bellville washer general loading case

**Figure 4** – Schematic of damper test rig

**Figure 5** – Pressure-Flow curve for a single type A washer at different torque settings

**Figure 6** – Pressure-Flow curve for type B washers stacked in parallel

**Figure 7** – Method used to predict washer pressure/flow characteristic

**Figure 8** – Comparison between simulated and experimental pressure-flow characteristics for a single type A

**Figure 9** – Predicted and measured type A washer load-deflection curves

**Figure 10** – Variation in flow coefficients with flow number

**Figure 11** – CFD predicted velocities

**Figure 12** – Velocity profile at washer exit at 0.1mm deflection

**Figure 13** – Velocity profile at washer exit at 0.3mm deflection

**Figure 14** – Static and dynamic pressure variations along washer radius

**Figure 15** – Comparison between predicted and measured pressure/flow characteristics

**Figure 16** – Comparison between predicted and measured pressure/flow characteristic for parallel stacked washers

**Table 1** – Belleville washer dimensional data

	Washer A - mm	Washer B - mm
Outer radius – $a$	34.00	34.00
Inner radius – $b$	12.30	12.00
Thickness – $t$	1.00	1.25
Free height – $h$	2.20	2.45

**Table 2** - Predicted pressure differential (bar) obtained from CFD simulations

Flow Rate (L/min)	Washer Deflection (mm)				
	0.1	0.3	0.5	0.8	1.0
20	6.28	0.829	0.33	0.126	0.097
40	22.6	2.89	1.11	0.461	0.342
60	48.9	6.22	2.37	0.972	0.618
80	84.8	10.8	4.11	1.68	1.06

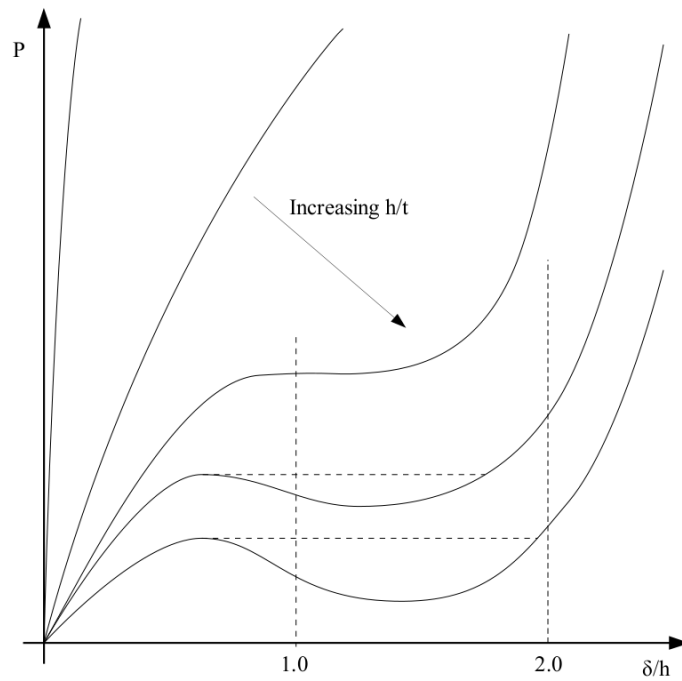
**Table 3** – Parametric data for type A washer

<i>Torque setting</i> $Nm$	$r_{eq1}$ $mm$	$k$	<i>Pre-load</i> $N$
1	11	2.2	400
2	11	1.8	850
3	12	0.75	1180
4	13	0.8	1400

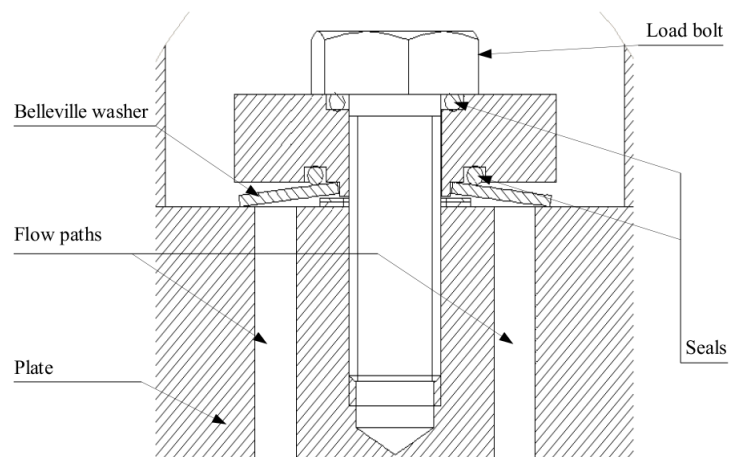


**Table 4** – Parametric data for type B washers

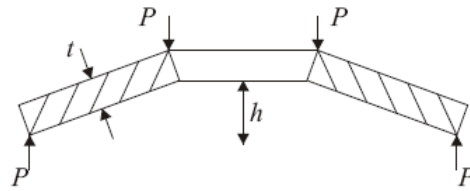
<i>Number of washers</i>	<i><math>r_{eq1}</math> mm</i>	<i><math>k</math></i>	<i>Pre-load <math>N</math></i>
1	11	1.5	3300
2	11	1.3	3000
3	13	0.65	2250



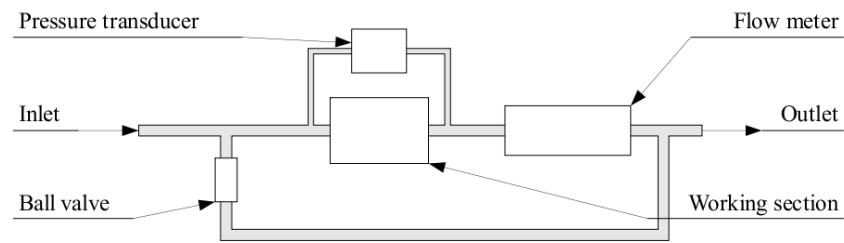
**Figure 1** – Effect of washer free height/thickness ratio on Belleville on load-deflection characteristic



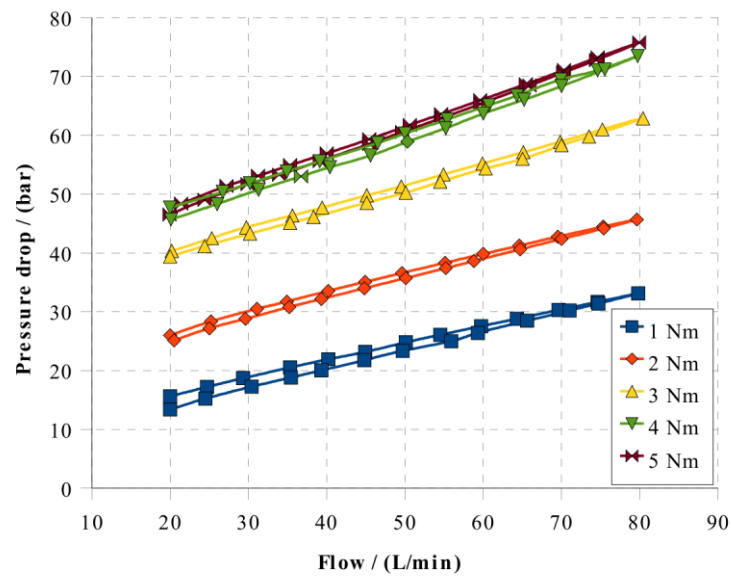
**Figure 2** - Disc spring used as a damper element



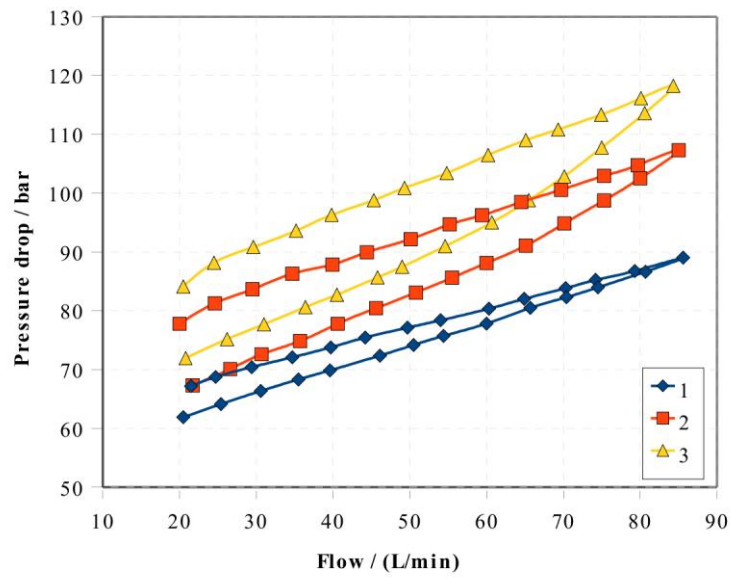
**Figure 3** – Bellville washer general loading case



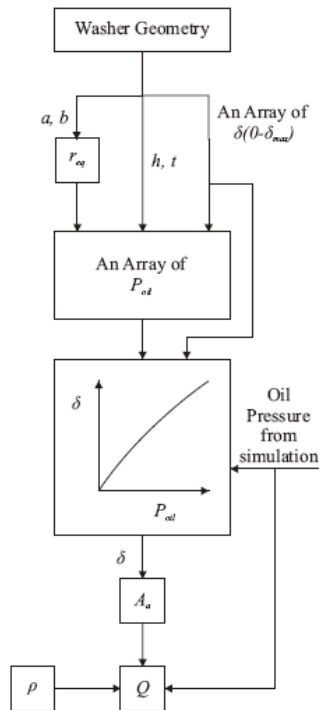
**Figure 4** – Schematic of damper test rig



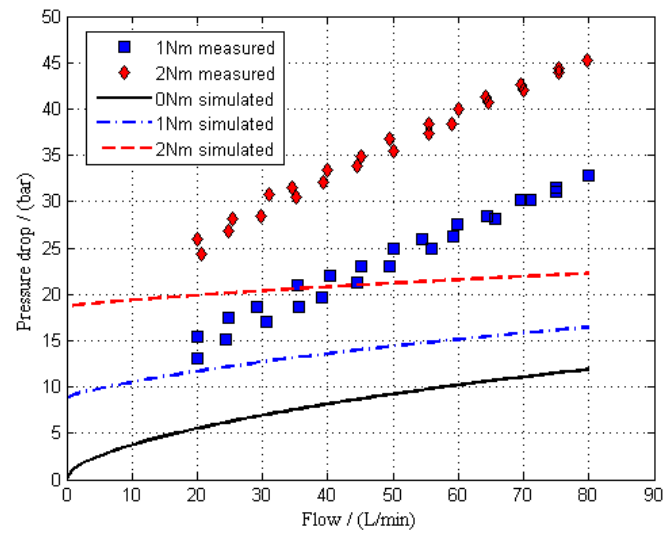
**Figure 5** – Pressure-Flow curve for a single type A washer at different torque settings



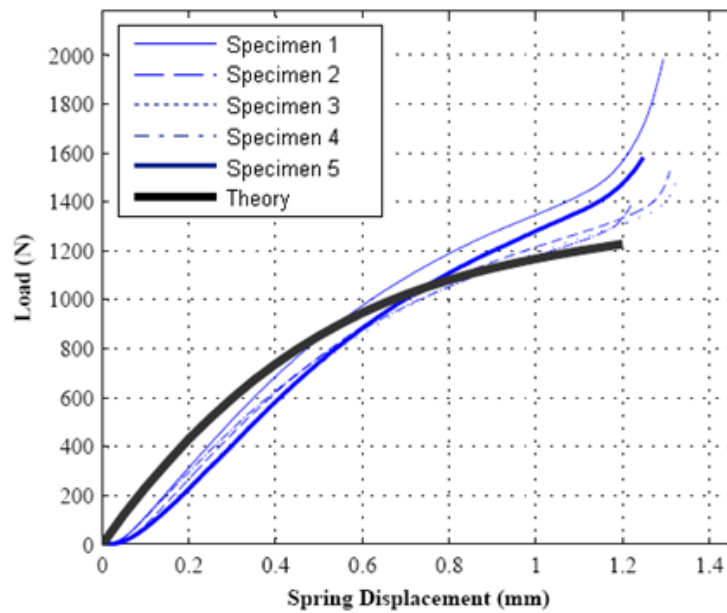
**Figure 6** – Pressure-Flow curve for type B washers stacked in parallel



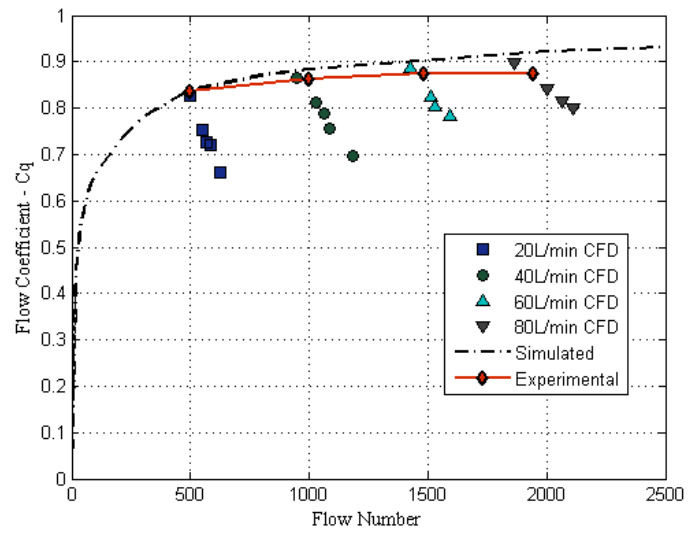
**Figure 7** – Method used to predict washer pressure/flow characteristic



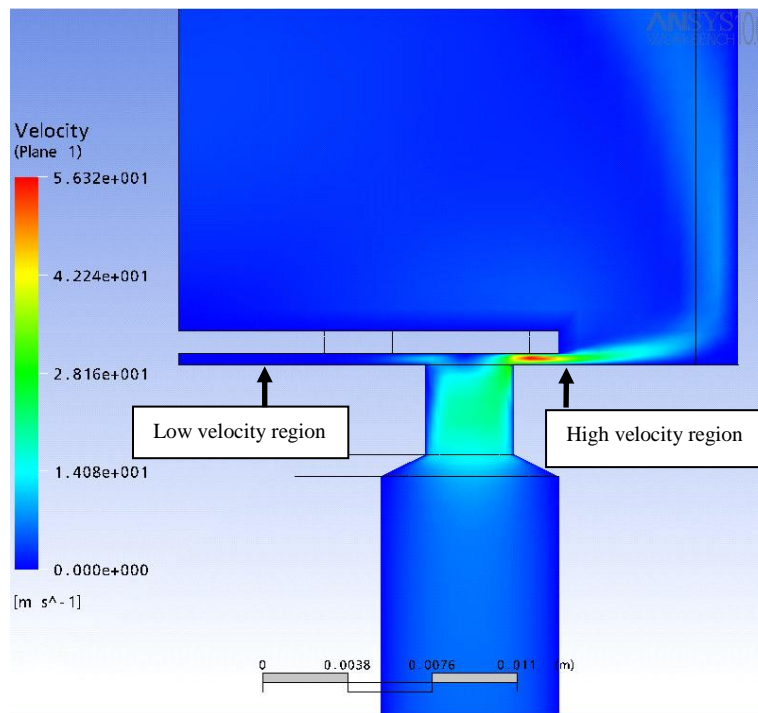
**Figure 8** – Comparison between simulated and experimental pressure-flow characteristics for a single type A



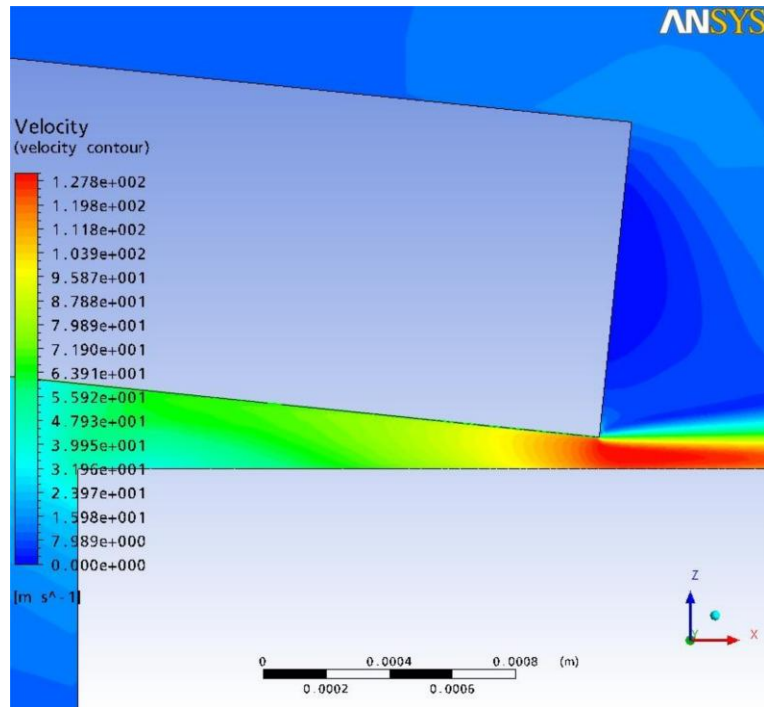
**Figure 9** – Predicted and measured type A washer load-deflection curves



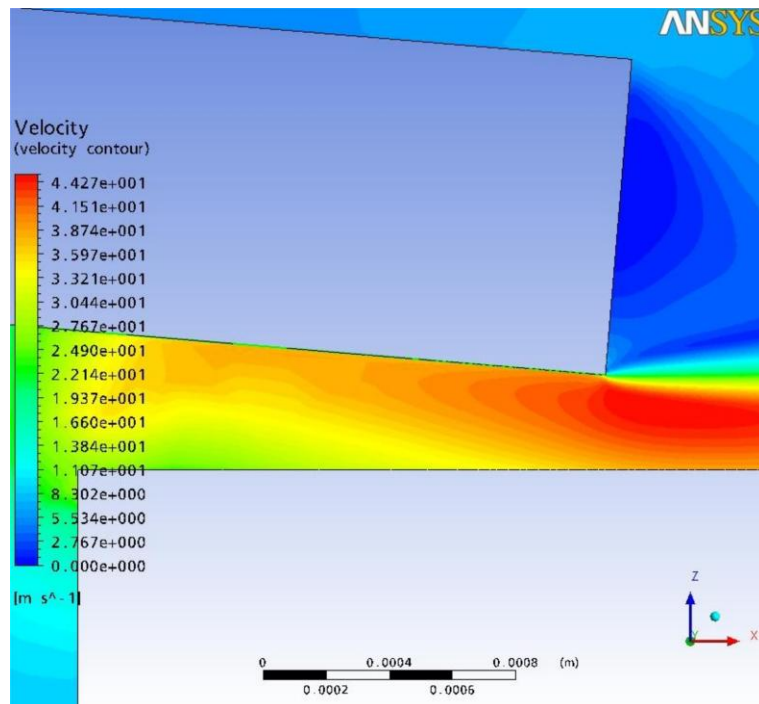
**Figure 10** – Variation in flow coefficients with flow number



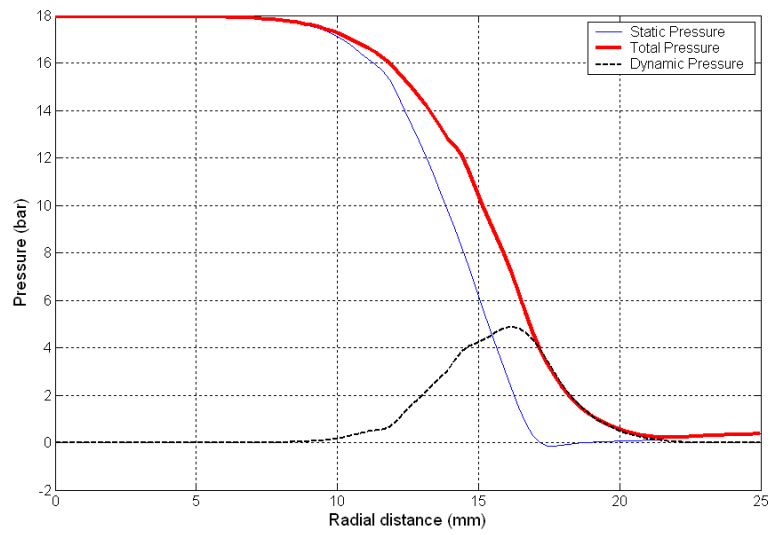
**Figure 11** – CFD predicted velocities



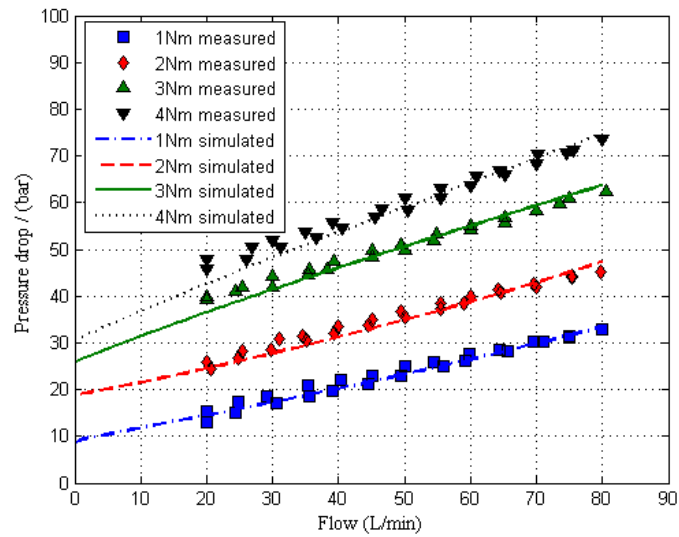
**Figure 12** – Velocity profile at washer exit at 0.1mm deflection



**Figure 13** – Velocity profile at washer exit at 0.3mm deflection

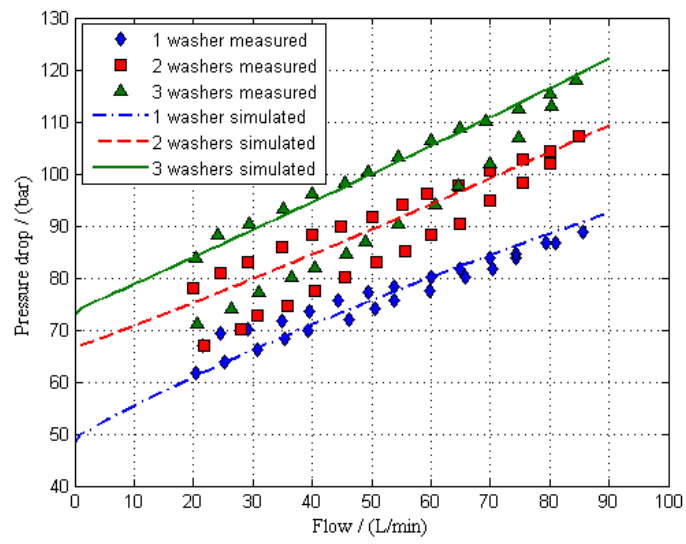


**Figure 14** – Static and dynamic pressure variations along washer radius



**Figure 15** – Comparison between predicted and measured pressure/flow characteristics





**Figure 16** – Comparison between predicted and measured pressure/flow characteristic for parallel stacked washers